On the Semantics of Simple Contrapositive Assumption-Based Argumentation Frameworks

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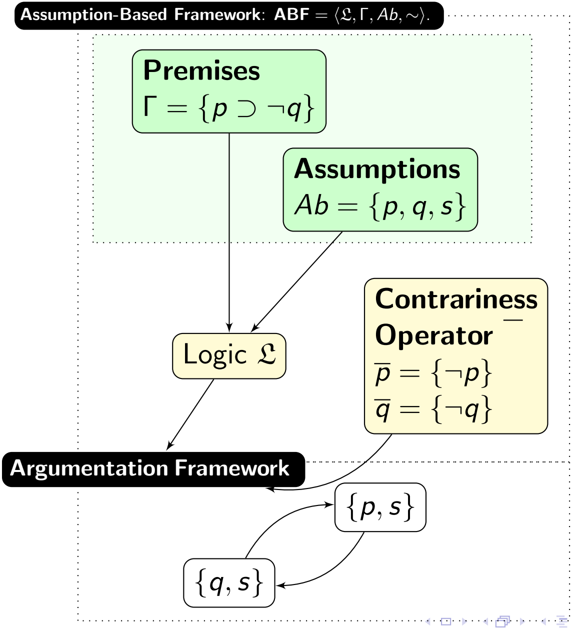
CLAR 2018

[1]Supported by the Sofja Kovalevskaja award of the Alexander von Humboldt-Foundation, funded by the German Ministry for Education and Research

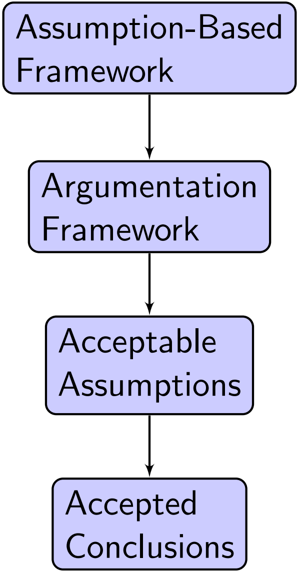
[2]Supported by the Israel Science Foundation (grant number 817/15).

The Plan

Assumption-Based Argumentation



The Argumentation Pipeline



Logic

Definition

A (propositional) *logic* for a language $\mathcal{L}$ is a pair $\mathfrak{L} = \langle \mathcal{L}, \vdash \rangle$, where $\vdash$ is a consequence relation for $\mathcal{L}$ satisfying the following conditions:

- *Reflexivity*: if $\psi \in \Gamma$ then $\Gamma \vdash \psi$.
- *Monotonicity*: if $\Gamma \vdash \psi$ and $\Gamma \subseteq \Gamma'$, then $\Gamma' \vdash \psi$.
- *Transitivity*: if $\Gamma \vdash \psi$ and $\Gamma', \psi \vdash \phi$ then $\Gamma, \Gamma' \vdash \phi$.

Connectives

Definition

We shall assume that the language $\mathcal{L}$ contains at least the following connectives:

- a $\vdash$-*negation* $\neg$, satisfying: $p \nvdash \neg p$ and $\neg p \nvdash p$ (for every atomic p)
- a $\vdash$-*conjunction* $\wedge$, satisfying: $\Gamma \vdash \psi \wedge \phi$ iff $\Gamma \vdash \psi$ and $\Gamma \vdash \phi$
- a $\vdash$-*disjunction* $\vee$, satisfying: $\Gamma, \phi \vee \psi \vdash \sigma$ iff $\Gamma, \phi \vdash \sigma$ and $\Gamma, \psi \vdash \sigma$
- a $\vdash$-*implication* $\supset$, satisfying: $\Gamma, \phi \vdash \psi$ iff $\Gamma \vdash \phi \supset \psi$.
- a $\vdash$-*falsity* F, satisfying: $\mathsf{F} \vdash \psi$ for every formula ψ.

Some Conditions on Logics

Definition

- Θ *is* $\vdash$*-inconsistent if* $\Theta \vdash \mathsf{F}$.
- *A logic* $\mathfrak{L} = \langle \mathcal{L}, \vdash \rangle$ *is explosive, if for every* $\psi \in \mathcal{L}$*, the set* $\{\psi, \neg\psi\}$ *is* $\vdash$*-inconsistent.*
- *We say that* $\mathfrak{L}$ *is contrapositive, if for every* $\Gamma \cup \{\psi\} \subseteq \mathcal{L}$ *it holds that:*
  $\Gamma \vdash \neg\psi$ *iff:*
  - $\psi = \mathsf{F}$, *or*
  - *for every* $\phi \in \Gamma$ *we have that* $\Gamma \setminus \{\phi\}, \psi \vdash \neg\phi$.

Assumption-based framework

Definition

An *assumption-based framework* is a tuple $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$ where:

- $\mathfrak{L} = \langle \mathcal{L}, \vdash \rangle$ is a propositional Tarskian logic
- Γ (the *strict assumptions*) is a $\vdash$-consistent set of $\mathcal{L}$-formulas, and
- Ab (the *candidate/defeasible assumptions*)(assumed to be nonempty),
- $\sim : Ab \to \wp(\mathcal{L})$ is a *contrariness operator* (such that for every $\psi \in Ab \setminus \{\mathsf{F}\}$ it holds that $\psi \nvdash \bigwedge \sim \psi$ and $\bigwedge \sim \psi \nvdash \psi$).

Simple Contrapositive ABF

Definition

A simple contrapositive *ABF is an assumption-based framework* $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$, *where:*

- $\mathfrak{L}$ *is an explosive and contrapositive logic, and,*
- $\sim\psi = \{\neg\psi\}$.

Attacks

Definition

Let $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$ *be an assumption-based framework,* $\Delta, \Theta \subseteq Ab$*, and* $\psi \in Ab$*.*

- Δ *attacks* ψ *iff* $\Gamma, \Delta \vdash \phi$ *for some* $\phi \in \sim\psi$*.*
- Δ *attacks* Θ *if* Δ *attacks some* $\psi \in \Theta$*.*

Example

Example

Let $\mathfrak{L} = \mathrm{CL}$, $\Gamma = \{p \supset \neg s\}$, *and* $Ab = \{p, s, t\}$.

$$\{s\} \qquad \{p\} \qquad \{t\}$$

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Let $\mathfrak{L} = \mathsf{CL}$, $\Gamma = \{p \supset \neg s\}$, *and* $Ab = \{p, s, t\}$.

$$\{s\} \longleftrightarrow \{p\} \qquad \qquad \{t\}$$

Argumentation Semantics

Definition ([3])

Let $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$ *and* $\Delta \subseteq Ab$. *We say that:*

- Δ *is closed if* $\Delta = Ab \cap Cn_{\vdash}(\Gamma \cup \Delta)$.
- Δ *is conflict-free iff there is no* $\Delta' \subseteq \Delta$ *that attacks* Δ.
- Δ *is naive iff it is closed and maximally conflict-free.*
- Δ *defends a set* $\Delta' \subseteq Ab$ *iff for every closed set* Θ *that attacks* Δ' *there is* $\Delta'' \subseteq \Delta$ *that attacks* Θ.
- Δ *is admissible iff it is closed, conflict-free, and defends every* $\Delta' \subseteq \Delta$.
- Δ *is complete iff it is admissible and contains every* $\Delta' \subseteq Ab$ *that it defends.*
- Δ *is grounded iff it is minimally complete.*
- Δ *is preferred iff it is maximally admissible.*
- Δ *is stable iff it is closed, conflict-free, and attacks every* $\psi \in Ab \setminus \Delta$.

Argumentation Semantics

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Let $\mathfrak{L} = \mathsf{CL}$, $\Gamma = \{p \supset \neg s; p \supset t\}$, *and* $Ab = \{p, s, t\}$.

$$\{s\} \longleftrightarrow \{p\} \qquad\qquad \{t\}$$

$\emptyset$, $\{s\}$, $\{t\}$, $\{p, t\}$ *and* $\{s, t\}$ *are admissible.*
$\{p\}$ *is not admissible since it is not closed.*

Example

Example

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$$\{s\} \longleftrightarrow \{p\} \qquad\qquad \{t\}$$

$\{t\}$, $\{p, t\}$ *and* $\{s, t\}$ *are complete.*

Example

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Let $\mathfrak{L} = \mathsf{CL}$, $\Gamma = \{p \supset \neg s; p \supset t\}$, *and* $Ab = \{p, s, t\}$.

$$\{s\} \longleftrightarrow \{p\} \qquad \{t\}$$

$\{p, t\}$ *and* $\{s, t\}$ *are preferred (and stable).*

Example

Example

Let $\mathfrak{L} = \mathrm{CL}$, $\Gamma = \{p \supset \neg s; p \supset t\}$, *and* $Ab = \{p, s, t\}$.

$$\{s\} \longleftrightarrow \{p\} \qquad\qquad \{t\}$$

$\{t\}$ *is grounded.*

Entailment

Definition

Given an assumption-based framework $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$*. For* $\mathsf{Sem} \in \{\mathsf{Grd}, \mathsf{Prf}, \mathsf{Stb}\}$*, we denote:*

- $\mathbf{ABF} \mid\!\sim^{\cap}_{\mathsf{Sem}} \psi$ *iff* $\Gamma, \Delta \vdash \psi$ *for every* $\Delta \in \mathsf{Sem}(\mathbf{ABF})$*.*
- $\mathbf{ABF} \mid\!\sim^{\cup}_{\mathsf{Sem}} \psi$ *iff* $\Gamma, \Delta \vdash \psi$ *for some* $\Delta \in \mathsf{Sem}(\mathbf{ABF})$*.*

Where $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$, we will also sometimes say that $\Gamma, Ab \mid\!\sim^{\star}_{\mathsf{Sem}} \psi$ if $\mathbf{ABF} \mid\!\sim_{\mathsf{Sem}} \star\, \psi$ (for some $\star \in \{\cap, \cup\}$).

Preferential and Stable Semantics

Preferential and Stable Semantics

Proposition

Let $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$ *be a simple contrapositive ABF and* $\Delta \subseteq Ab$*. Then:*

$$\Delta \textit{ is naive} \quad \textit{iff} \quad \Delta \textit{ is stable} \quad \textit{iff} \quad \Delta \textit{ is preferred.}$$

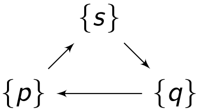
But why?

> *We [. . .] note that in every semi-stable labelling of an AF without stable labellings there exists an odd-length cycle whose arguments are all labelled undec [7]*

In other words, odd length cycles are in most cases responsible for preferred extensions not being stable.

Example

Suppose that $\mathfrak{L} = \mathsf{CL}$, $\Gamma = \{p \supset \neg s, s \supset \neg q, q \supset \neg p\}$, *and* $Ab = \{p, q, s\}$.



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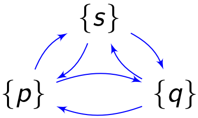


Relation with MCS

Definition

Let $\textbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$. A set $\Delta \subseteq Ab$ is *maximally consistent* in **ABF**, if

- $\Gamma, \Delta \nvdash \mathsf{F}$ and
- $\Gamma, \Delta' \vdash \mathsf{F}$ for every $\Delta \subsetneq \Delta' \subseteq Ab$.

The set of the maximally consistent sets in **ABF** is denoted $\mathsf{MCS}(\textbf{ABF})$.

Proposition

Let $\textbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$ *be a simple contrapositive ABF and* $\Delta \subseteq Ab$. *Then:*

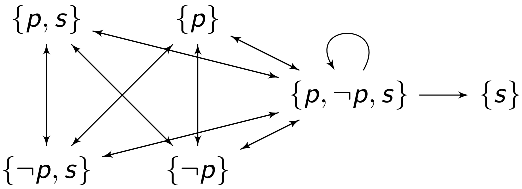
Δ *is naive iff* Δ *is stable iff* Δ *is preferred iff* $\Delta \in \mathsf{MCS}(\textbf{ABF})$

The Grounded Semantics

A Problematic Example

Example

Let $\mathfrak{L} = \mathsf{CL}$*,* $\Gamma = \emptyset$*, and* $Ab = \{p, \neg p, s\}$*.*

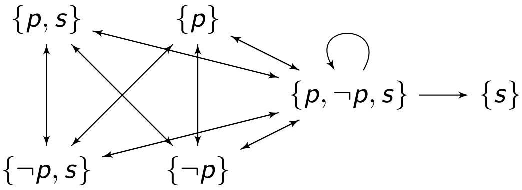


- s is an innocent bystander that is not derivable using the grounded extension.

A Problematic Example

Example

Let $\mathfrak{L} = \mathsf{CL}$, $\Gamma = \emptyset$, *and* $Ab = \{p, \neg p, s\}$.

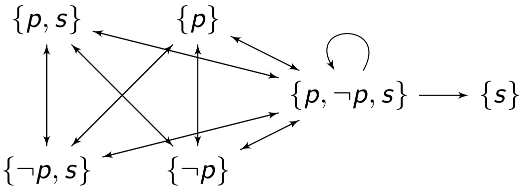


- s is an innocent bystander that is not derivable using the grounded extension.
- Contamination problems.

A Problematic Example

Example

Let $\mathfrak{L} = \mathsf{CL}$, $\Gamma = \emptyset$, *and* $Ab = \{p, \neg p, s\}$.



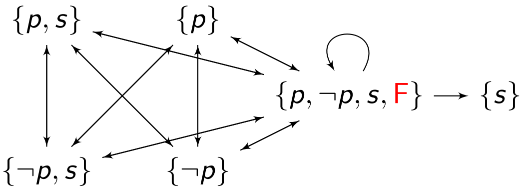
- s is an innocent bystander that is not derivable using the grounded extension.
- Contamination problems.
- No correspondence with maximal consistent subsets.

A simple solution

Add F to *Ab*.

Example (Example 4 continued)

Let $\mathfrak{L} = \mathsf{CL}$, $\Gamma = \emptyset$, *and* $Ab = \{p, \neg p, s, \mathsf{F}\}$.



A simple solution

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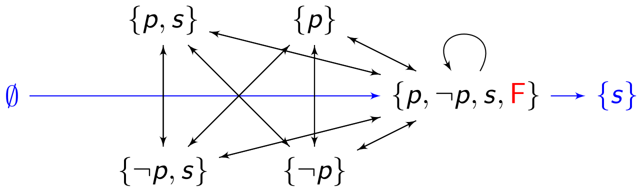
Example (Example 4 continued)

Let $\mathfrak{L} = \mathrm{CL}$, $\Gamma = \emptyset$, *and* $Ab = \{p, \neg p, s, \mathrm{F}\}$.



In Fact:

Theorem

Let $\mathbf{ABF} = \langle \mathfrak{L}, \Gamma, Ab, \sim \rangle$ *be a simple contrapositive assumption-based framework in which* $\mathsf{F} \in Ab$*. Then* $\mathsf{Grd}(\mathbf{ABF}) = \bigcap \mathsf{MCS}(\mathbf{ABF})$*.*

Interference

Definition

Given a logic $\mathfrak{L} = \langle \mathcal{L}, \vdash \rangle$*, let* Γ_i $(i = 1, 2)$ *be two sets of* $\mathcal{L}$*-formulas, and let* $\mathbf{ABF}_i = \langle \mathfrak{L}, \Gamma_i, Ab_i, \sim_i \rangle$ $(i = 1, 2)$ *be two ABFs based on* $\mathfrak{L}$*.*

- *We denote by* $\mathrm{Atoms}(\Gamma_i)$ $(i = 1, 2)$ *the set of all atoms occurring in* Γ_i*.*
- *We say that* Γ_1 *and* Γ_2 *are* syntactically disjoint *if* $\mathrm{Atoms}(\Gamma_1) \cap \mathrm{Atoms}(\Gamma_2) = \emptyset$*.*
- $\mathbf{ABF}_1$ *and* $\mathbf{ABF}_2$ *are* syntactically disjoint *if so are* $\Gamma_1 \cup Ab_1$ *and* $\Gamma_2 \cup Ab_2$*.*
- *We denote:* $\mathbf{ABF}_1 \cup \mathbf{ABF}_2 = \langle \mathfrak{L}, \Gamma_1 \cup \Gamma_2, Ab_1 \cup Ab_2, \sim_1 \cup \sim_2 \rangle$*.*

An entailment $\mid\!\sim$ *satisfies* non-interference, *if for every two syntactically disjoint frameworks* $\mathbf{ABF}_1 = \langle \mathfrak{L}, \Gamma_1, Ab_1, \sim_1 \rangle$ *and* $\mathbf{ABF}_2 = \langle \mathfrak{L}, \Gamma_2, Ab_2, \sim_2 \rangle$ *where* $\Gamma_1 \cup \Gamma_2$ *is consistent, it holds that:* $\mathbf{ABF}_1 \mid\!\sim \psi$ *iff* $\mathbf{ABF}_1 \cup \mathbf{ABF}_2 \mid\!\sim \psi$ *for every* $\mathcal{L}$*-formula* ψ *s.t.* $\mathrm{Atoms}(\psi) \subseteq \mathrm{Atoms}(\Gamma_1 \cup Ab_1)$*.*

Non-Interference

Theorem

For Sem $\in \{\mathsf{Naive}, \mathsf{Prf}, \mathsf{Stb}\}$*, both* $\mathrel{|\!\sim}^{\cup}_{\mathsf{Sem}}$ *and* $\mathrel{|\!\sim}^{\cap}_{\mathsf{Sem}}$ *satisfy non-interference with respect to simple contrapositive assumption-based frameworks.*

Theorem

$\mathrel{|\!\sim}_{\mathsf{Grd}}$ *satisfies non-interference for any simple contrapositive ABF in which* $\mathsf{F} \in Ab$*.*

Preferential Entailments

KLM properties

Definition ([6])

A relation $\mid\!\sim$ *between ABFs and formulas is* cumulative, *if the following conditions are satisfied:*

- *Cautious Reflexivity (CR): For every* $\vdash$*-consistent* ψ *it holds that* $\psi \mid\!\sim \psi$
- *Cautious Monotonicity (CM): If* $\Gamma, Ab \mid\!\sim \phi$ *and* $\Gamma, Ab \mid\!\sim \psi$ *then* $\Gamma, Ab, \phi \mid\!\sim \psi$
- *Cautious Cut (CC): If* $\Gamma, Ab \mid\!\sim \phi$ *and* $\Gamma, Ab, \phi \mid\!\sim \psi$ *then* $\Gamma, Ab \mid\!\sim \psi$.
- *Left Logical Equivalence (LLE): If* $\phi \vdash \psi$ *and* $\psi \vdash \phi$ *then* $\Gamma, Ab, \phi \mid\!\sim \rho$ *iff* $\Gamma, Ab, \psi \mid\!\sim \rho$.
- *Right Weakening (RW): If* $\phi \vdash \psi$ *and* $\Gamma, Ab \mid\!\sim \phi$ *then* $\Gamma, Ab \mid\!\sim \psi$.

A cumulative relation is preferential, *if it satisfies:*

- *Distribution (OR): If* $\Gamma, Ab, \phi \mid\!\sim \rho$ *and* $\Gamma, Ab, \psi \mid\!\sim \rho$ *then* $\Gamma, Ab, \phi \vee \psi \mid\!\sim \rho$.

Results for Skeptical Entailments

Proposition

Let $\mathbf{ABF} = \langle \mathcal{L}, \Gamma, Ab, \sim \rangle$ *be a simple contrapositive ABF. Then* $\mid\!\sim^{\cap}_{\mathsf{Sem}}$ *is preferential for* $\mathsf{Sem} \in \{\mathsf{Naive}, \mathsf{Prf}, \mathsf{Stb}\}$.
If $\mathsf{F} \in Ab$*, then* $\mid\!\sim^{\cap}_{\mathsf{Grd}}$ *is also preferential.*

Results for Credulous Entailments

Example

Let $\mathfrak{L} = \mathsf{CL}$, $\Gamma = \emptyset$, *and* $Ab = \{r \wedge (q \supset p), \neg r \wedge (t \supset p)\}$.
Note that:

- $\mathsf{MCS}(\langle \mathcal{L}, \Gamma, Ab \cup \{q\}, \sim\rangle) = \{\{r \wedge (q \supset p), q\}, \{\neg r \wedge (t \supset p), q\}\}$
- $\mathsf{MCS}(\langle \mathcal{L}, \Gamma, Ab \cup \{t\}, \sim\rangle) = \{\{r \wedge (q \supset p), t\}, \{\neg r \wedge (t \supset p), t\}\}$
- $\mathsf{MCS}(\langle \mathcal{L}, \Gamma, Ab \cup \{q \vee t\}, \sim\rangle) = \{\{r \wedge (q \supset p), q \vee t\}, \{\neg r \wedge (t \supset p), q \vee t\}\}$

Then $Ab, q \mid\!\sim p$ *and* $Ab, t \mid\!\sim p$ *but* $Ab, q \vee t \not\mid\!\sim p$ *for every entailment of the form* $\mid\!\sim^{\cup}_{\mathsf{Sem}}$ *where* $\mathsf{Sem} \in \{\mathsf{Naive}, \mathsf{Prf}, \mathsf{Stb}\}$.

Proposition

Let $\mathbf{ABF} = \langle \mathcal{L}, \Gamma, Ab, \sim\rangle$ *be a simple contrapositive ABF. Then* $\mid\!\sim^{\cup}_{\mathsf{Sem}}$ *is cumulative for* $\mathsf{Sem} \in \{\mathsf{Naive}, \mathsf{Prf}, \mathsf{Stb}\}$.

Discussion (in view of related work)

- Much work has been done on classical respectively Tarskian logic instantiations of Dung argumentation [1, 2, 4, 8].
- However, for ABA such a study was missing.
- For grounded semantics, some care has to be taken (similar problems have been discussed in [5]).
- Other semantics work as expected.
- A benefit of ABA is that for a finite knowledge base we obtain a finite argumentation graph (which is not the case for many other formalisms).

Future work

- Disjunctive attacks (e.g. $\{\neg p \vee \neg q\}$ attacks $\{p, q\}$).
- Closure requirements (turn out to be redundant).
- Modal Logics.

Thank you for your attention
Questions?

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